SN P Systems with Multiple Channels and Autapses

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The Catalogue

- SN P Systems with Multiple Channels (SNP-MC)
  - Small Universality of SNP-MC
  - SN P Systems with Multiple Channels and Anti-spikes
  - Asynchronous SNP-MC
- SN P Systems with Autapses

SN P Systems with Multiple Channels[1]

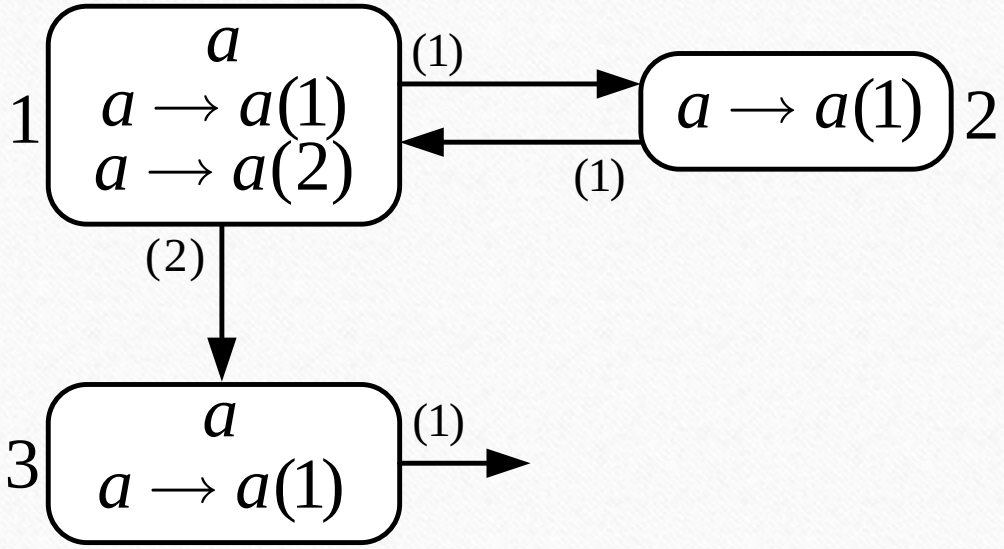
$$\Pi = (O, L, \sigma_1, \sigma_2, ..., \sigma_m, syn, in, out)$$

- (1) $O = \{a\}$ is the alphabet;
- (2) $L = \{1, 2, ...,N\}$ is channel labels;
- (3) $\sigma_1, \sigma_2, ..., \sigma_m$ are neurons of the form $\sigma_i = (n_i,R_i)$,
  - (a) $n_i \geq 0$ is the initial number of spikes contained in the neuron σ_i;
  - (b) R_i: $E/a^c \rightarrow a^p(l)$, $c \geq p \geq 1$, $l \in L_i$, $L_i \subseteq L$;
- (4) $syn \subseteq \{1, 2, ...,m\} \times \{1, 2, ...,m\} \times L$;
- (5) *in* and *out* indicate the input neuron and output neuron respectively.

[1] Hong Peng, Jinyu Yang, Jun Wang, Tao Wang, Zhang Sun, **Xiaoxiao Song**, Xiaohui Luo, Xiangnian Huang. Spiking neural P systems with multiple channels. Neural Networks, 95 (2017) 66-71.

SN P Systems with Multiple Channels

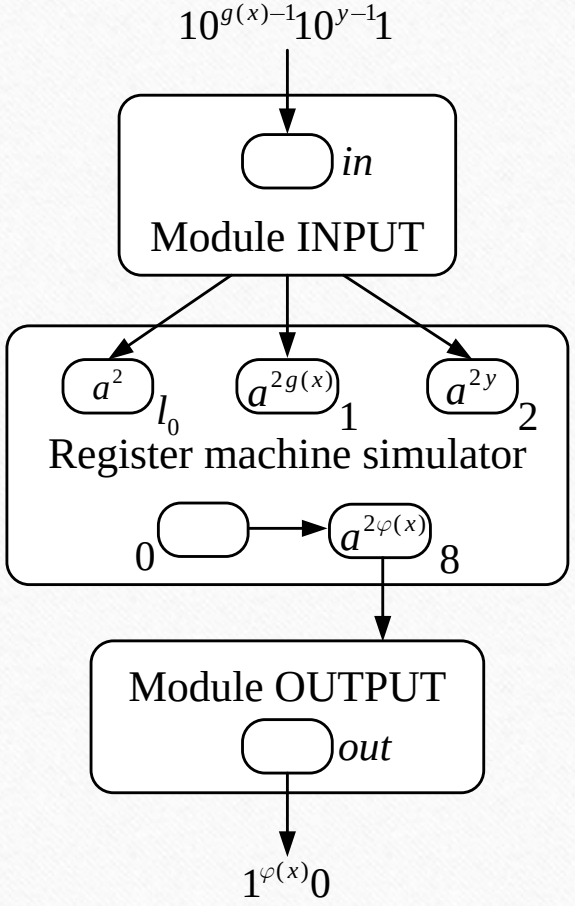
- An Example



- Result: 1(00)*1

Small Universality of SNP-MC[2]

- We obtain two universal systems with 57 neurons using standard spiking rules and 39 neurons using extended spiking rules for computing functions, respectively.
- *2n* spikes are used to represent number *n* in the registers.



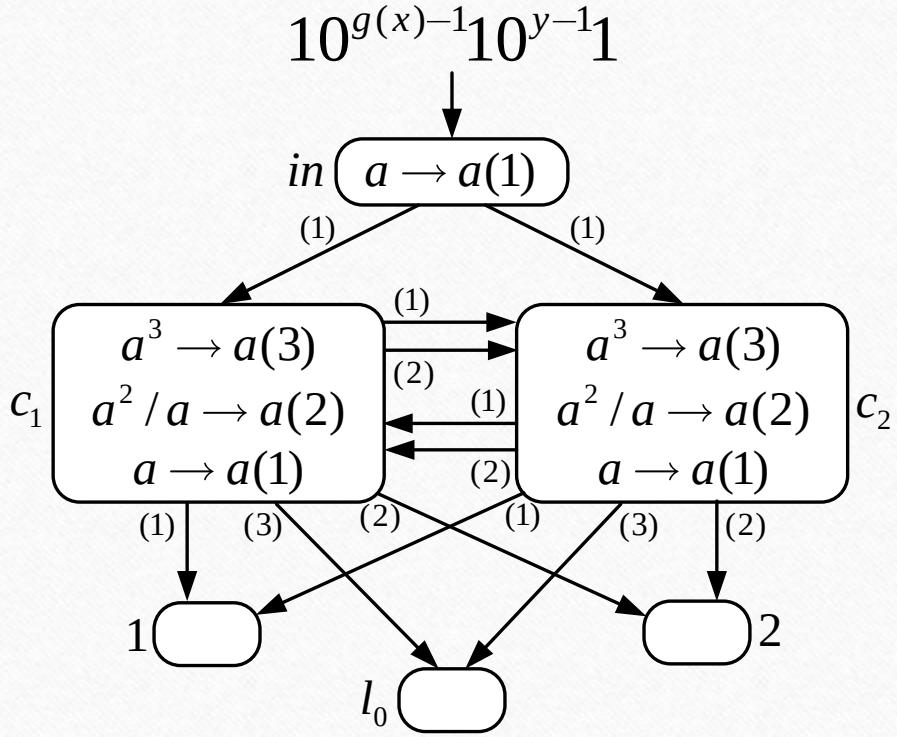
[2] **Xiaoxiao Song**, Hong Peng, Jun Wang, Guimin Ning, Tao Wang, Zhang Sun, Yankun Xia. On Small Universality of Spiking Neural P Systems with Multiple Channel. In book: Fuzzy Geometric Programming Techniques and Applications, pp.229-245, 10.1007/978-3-030-12797-8_16.

Small Universality of SNP-MC

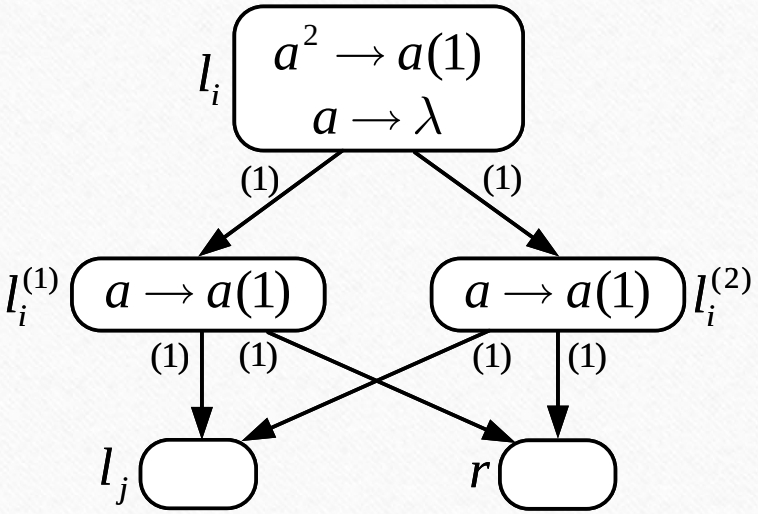
- Based on the universal register machine proposed by I. Korec (1996) and adjusted by G. Păun (2007).

$l_0 : (SUB(1),\ l_1,\ l_2)$, $l_1 : (ADD(7),\ l_0)$,
$l_2 : (ADD(6),\ l_3)$, $l_3 : (SUB(5),\ l_2,\ l_4)$,
$l_4 : (SUB(6),\ l_5,\ l_3)$, $l_5 : (ADD(5),\ l_6)$,
$l_6 : (SUB(7),\ l_7,\ l_8)$, $l_7 : (ADD(1),\ l_4)$,
$l_8 : (SUB(6),\ l_9,\ l_0)$, $l_9 : (ADD(6),\ l_{10})$,
$l_{10} : (SUB(4),\ l_0,\ l_{11})$, $l_{11} : (SUB(5),\ l_{12},\ l_{13})$,
$l_{12} : (SUB(5),\ l_{14},\ l_{15})$, $l_{13} : (SUB(2),\ l_{18},\ l_{19})$,
$l_{14} : (SUB(5),\ l_{16},\ l_{17})$, $l_{15} : (SUB(3),\ l_{18},\ l_{20})$,
$l_{16} : (ADD(4),\ l_{11})$, $l_{17} : (ADD(2),\ l_{21})$,
$l_{18} : (SUB(4),\ l_0,\ l_{22})$, $l_{19} : (SUB(0),\ l_0,\ l_{18})$,
$l_{20} : (ADD(0),\ l_0)$, $l_{21} : (ADD(3),\ l_{18})$,
$l_{22} : (SUB(0),\ l_{23},\ l_h')$, $l_{23} : (ADD(8),\ l_{22})$,
$l_h' : HALT$

SNP-MC Using Standard Spiking Rules



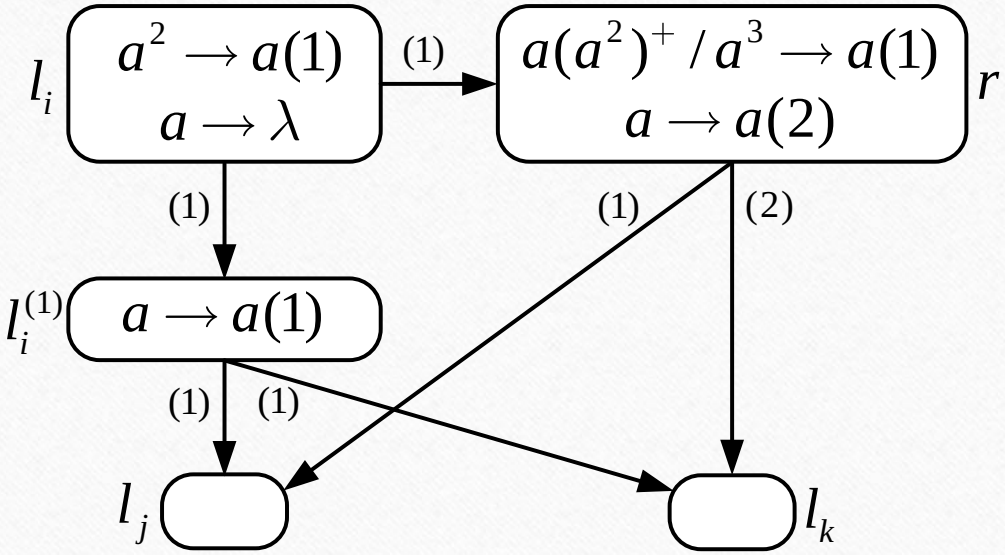
INPUT



ADD

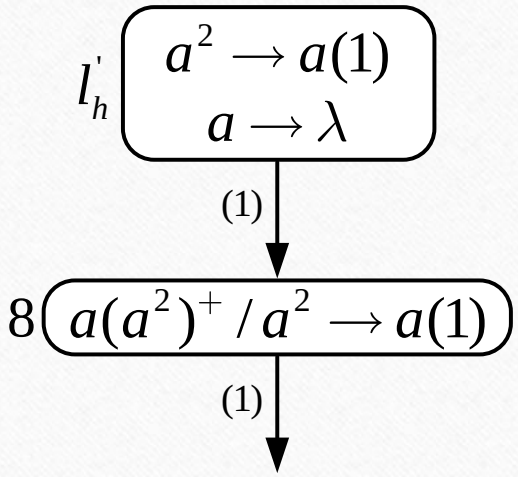
$l_i : (ADD(r),\ l_j)$

SNP-MC Using Standard Spiking Rules



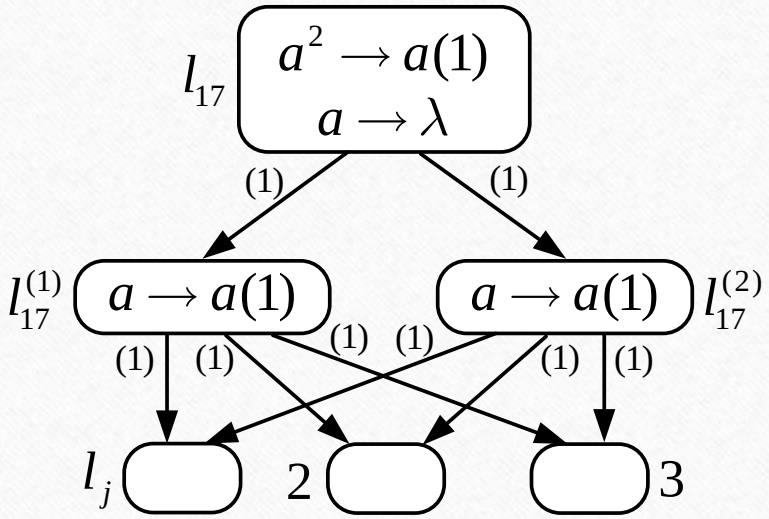
SUB

$l_i : (SUB(r), l_j, l_k)$



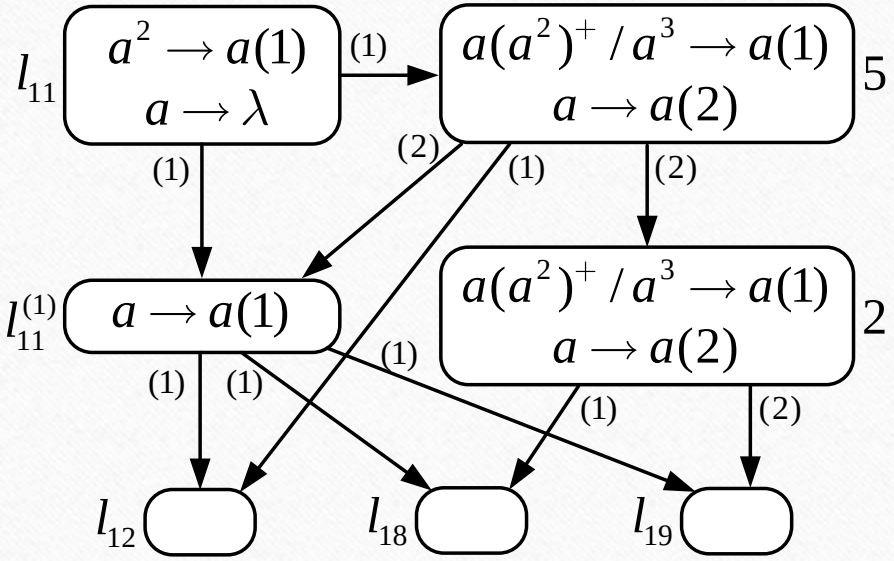
OUTPUT

SNP-MC Using Standard Spiking Rules



ADD-ADD

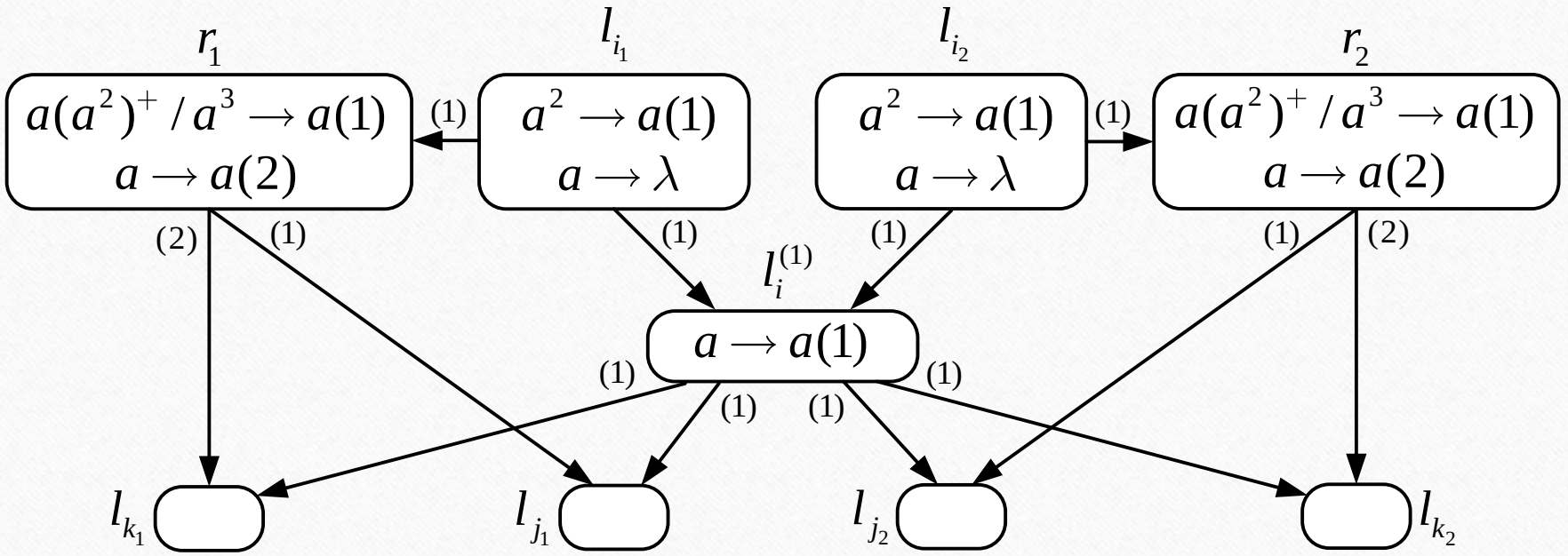
$l_{17} : (ADD(2), l_{21}), l_{21} : (ADD(3), l_{18})$



SUB-SUB

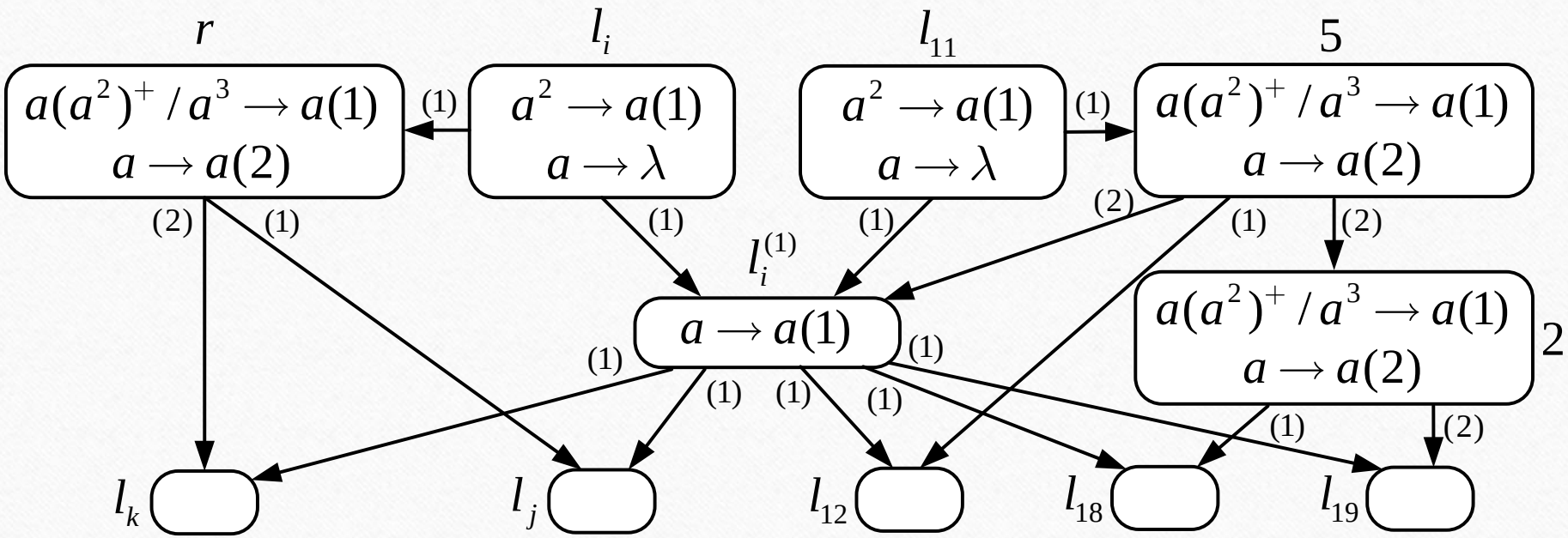
$l_{11} : (SUB(5), l_{12}, l_{13}), l_{13} : (SUB(2), l_{18}, l_{19})$

SNP-MC Using Standard Spiking Rules



Instructions l_0, l_3, l_4, l_6, l_{10}, l_{15} and l_{19}, which address registers 1, 5, 6, 7, 4, 3 and 0 respectively, can share a common neuron; instructions l_{10}, l_{15} and l_{19}, can also share a common neuron.

SNP-MC Using Standard Spiking Rules



A module simulating instructions l_{11} and l_{13} with other instruction with r≠2 and r≠5

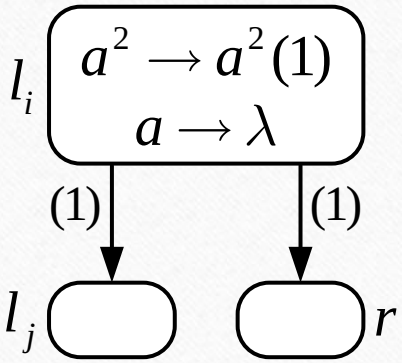
SNP-MC Using Standard Spiking Rules

- 9 neurons for 9 registers,
- 25 neurons for 25 instruction labels,
- 20 auxiliary neurons in ADD modules,
- 14 auxiliary neurons in SUB modules,
- 3 additional neurons in INPUT module,

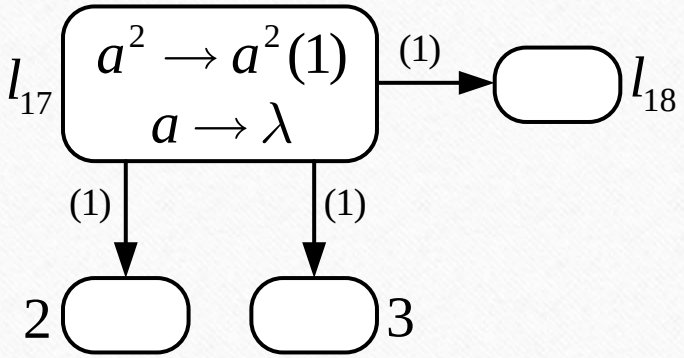
- which comes to a total of 71 neurons.
- By using the above optimization modules, we can totally save 14 neurons. The number of neurons can be decremented from 71 to 57.

SNP-MC Using Extended Spiking Rules

- The modules SUB, SUB-SUB, INPUT and OUTPUT are same with standard spiking rules.



ADD



ADD-ADD

- The number of neurons is 39.

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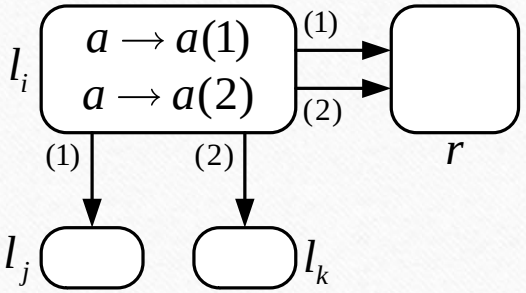
  - Small Universality of SNP-MC
  - SN P Systems with Multiple Channels and Anti-spikes
  - Asynchronous SNP-MC
- SN P Systems with Autapses

SN P Systems with Multiple Channels and Anti-spikes[3]

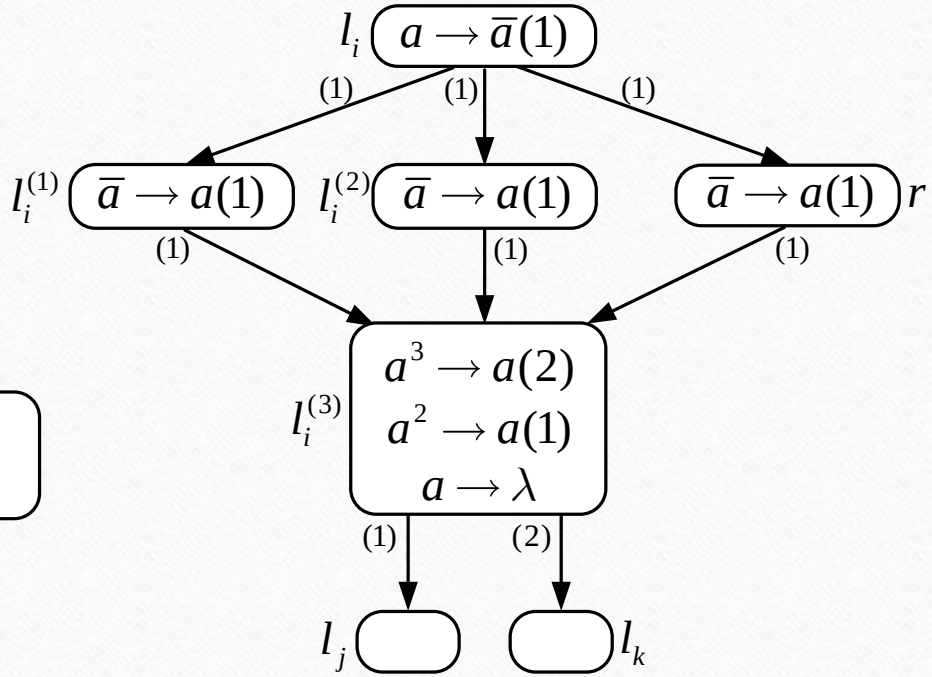
- SN P Systems with Multiple Channels and Anti-spikes (ASNP-MC)
  - The system can generate any recursively enumerable set of numbers;
  - There is a universal ASNP-MC system for computing functions having 65 neurons.
- $O = \{a, \bar{a}\}$ is the alphabet, where a and $\bar{a}$ are called spike and anti-spike respectively;
- When a neuron contains both spikes and anti-spikes, before any spiking rule and forgetting rule is used, $a\bar{a} \rightarrow \lambda$ is used at first in a maximal manner.
- n spikes are used to represent number n in the registers.

[3] **Xiaoxiao Song**, Jun Wang, Hong Peng, Guimin Ning, Zhang Sun, Tao Wang, Fan Yang. Spiking Neural P Systems with Multiple Channels and Anti-spikes. Biosystems, 2018, 169-170:13-19.

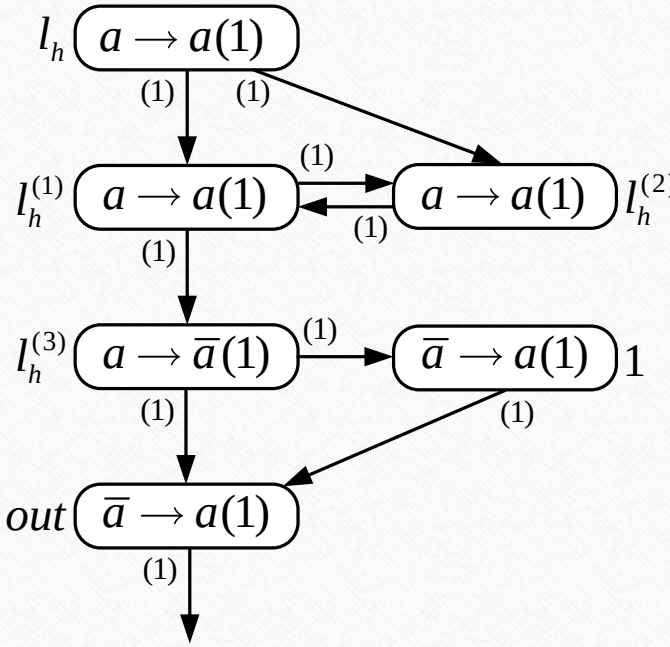
Generate Any Recursively Enumerable Set of Numbers



ADD

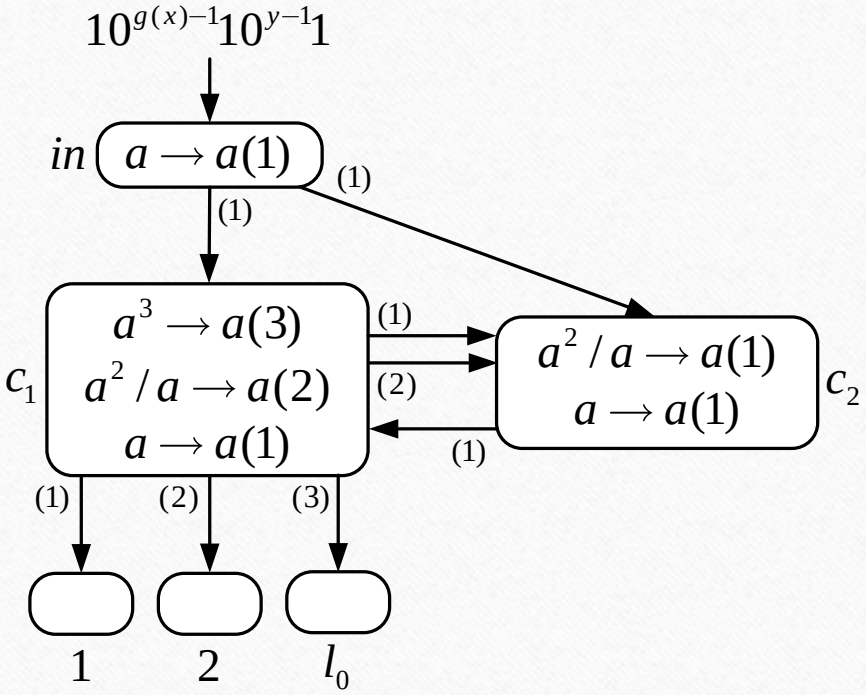


SUB

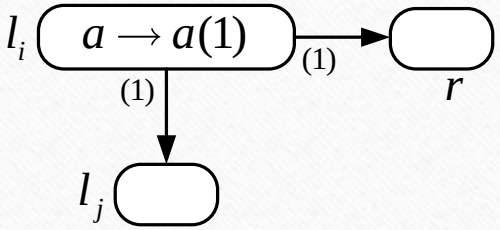


OUTPUT

Universal ASNP-MC Systems for Computing Functions



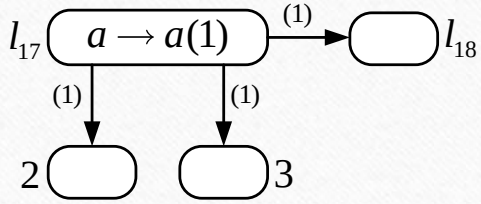
INPUT



ADD

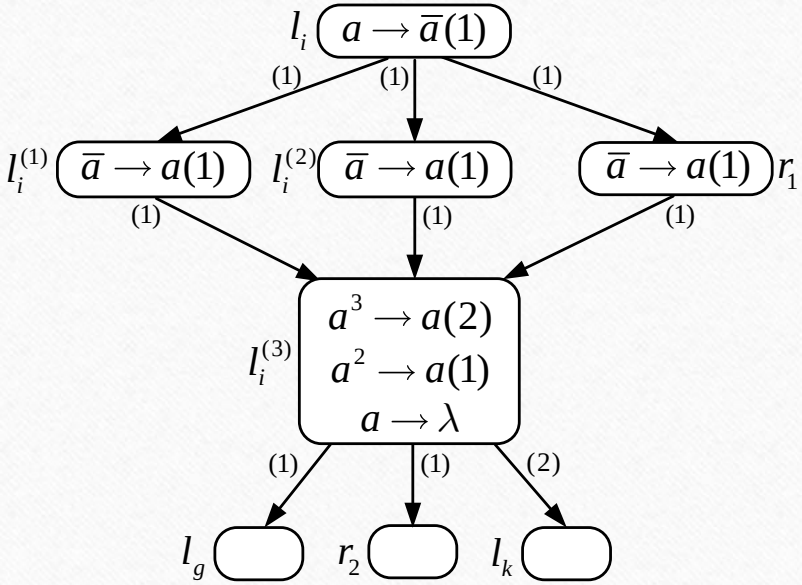
- SUB module and OUTPUT module are same as the above.

Universal ASNP-MC Systems for Computing Functions



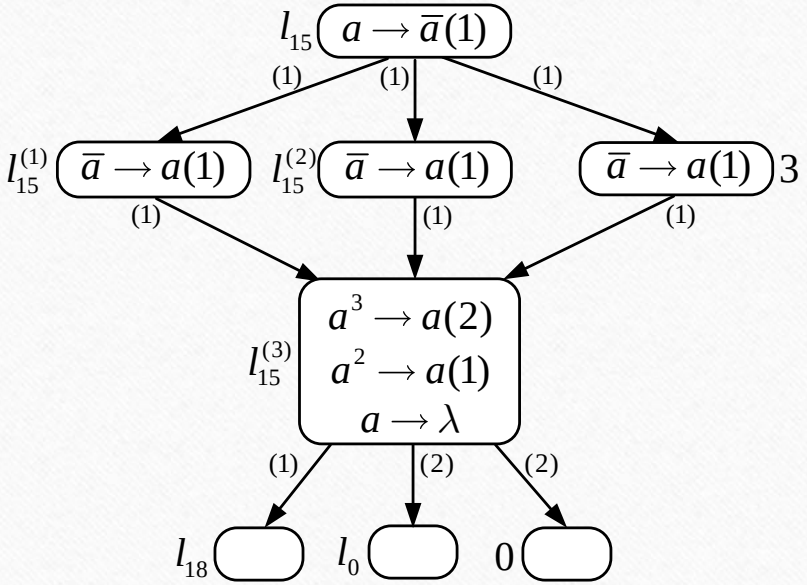
ADD-ADD

$l_{17} : (ADD(2), l_{21}), l_{21} : (ADD(3), l_{18})$



SUB-ADD-1

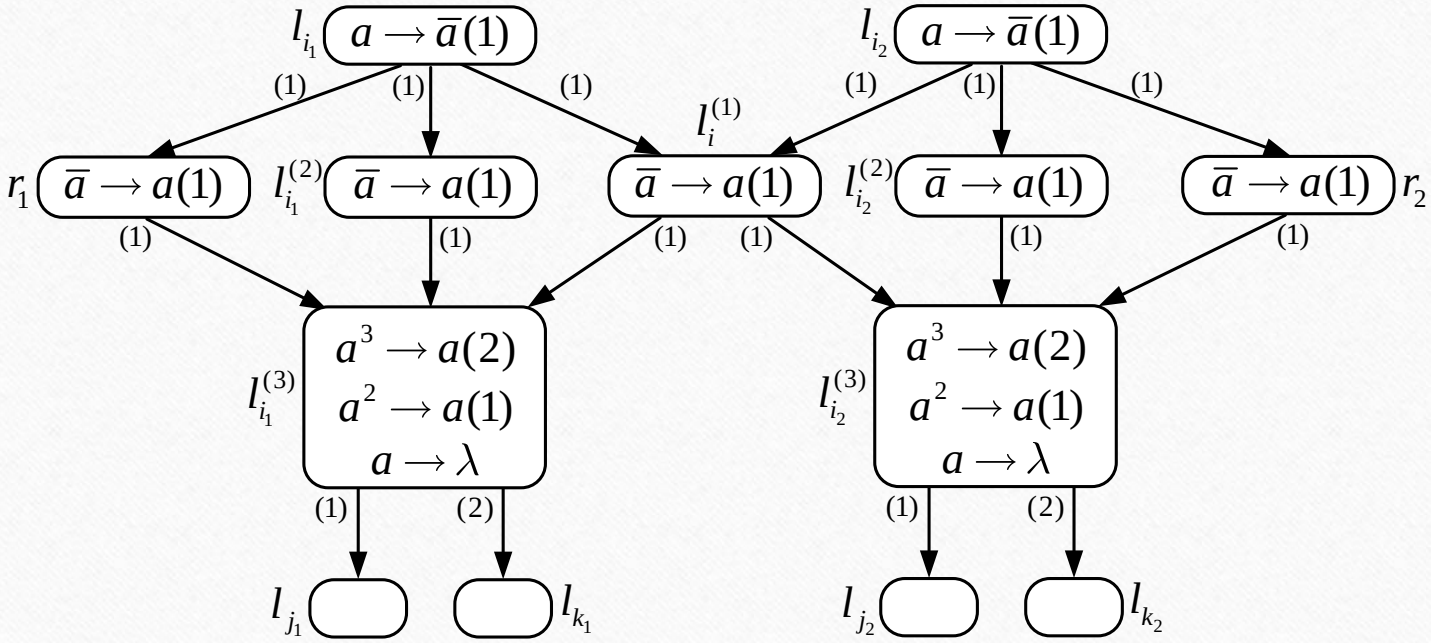
$l_i : (SUB(r_1), l_j, l_k), l_j : (ADD(r_2), l_g)$



SUB-ADD-2

$l_i : (SUB(r_1), l_j, l_k), l_k : (ADD(r_2), l_g)$

Universal ASNP-MC Systems for Computing Functions



- Instructions l_0, l_3, l_4, l_6, l_{10}, l_{13}, l_{15} and l_{19}, which address registers 1, 5, 6, 7, 4, 2, 3 and 0 respectively, can share a common neuron; instructions l_8, l_{11}, l_{18} and l_{22}, , which address registers 6, 5, 4 and 0 respectively, can share a common neuron.

Universal ASNP-MC Systems for Computing Functions

- 9 neurons for 9 registers,
- 25 neurons for 25 instruction labels,
- 42 auxiliary neurons in SUB modules,
- 3 additional neurons in INPUT module,
- 4 additional neurons in OUTPUT module,

- which comes to a total of 83 neurons.
- Totally, 18 neurons can be saved, and the number of neurons can be decremented from 84 to 66.

The Catalogue

- SN P Systems with Multiple Channels (SNP-MC)
  - Small Universality of SNP-MC
  - SN P Systems with Multiple Channels and Anti-spikes
  - Asynchronous SNP-MC
- SN P Systems with Autapses

Asynchronous SNP-MC System[4]

- Construct an asynchronous SNP-MC system using 38 neurons for computing functions;
- Construct an asynchronous SNP-MC system for generating numbers with 41 neurons.
- $3Tn$ spikes ($T=2max\{t_r|0\leq r\leq 7\}+1=2\cdot t_5+1=2\cdot 4+1=9$) are used to represent number n in the registers.

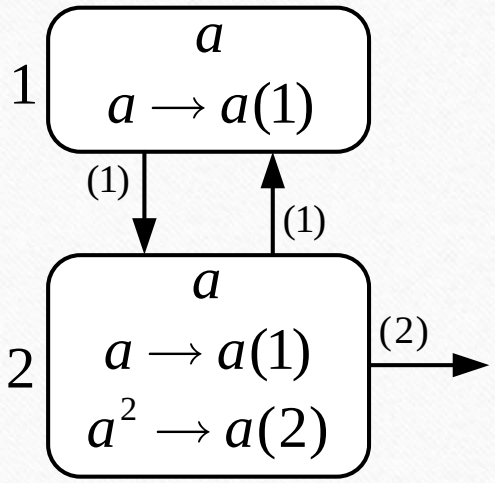
[4] **Xiaoxiao Song**, Hong Peng, Jun Wang, Guimin Ning, Zhang Sun. Small Universal Asynchronous Spiking Neural P Systems with Multiple Channels. Neurocomputing. After revised, under review for the second time.

Asynchronous SNP-MC System

- Asynchronous mode: **The use of enabled rules in neurons is not obligatory.** Neurons are free to use the rules or not. If the spikes stored in the neurons remain unchanged, the enabled rules can be applied in any later step. If other arriving spikes make the rules unusable, the computation continues under the new conditions.
- Synchronization plays a crucial role in the proof of the universality, which controls the computation of SN P systems. The computation power declines if the synchronization is removed from SN P systems. So more resource (e.g. number of neurons) are needed to compensate the loss in power.

Asynchronous SNP-MC System

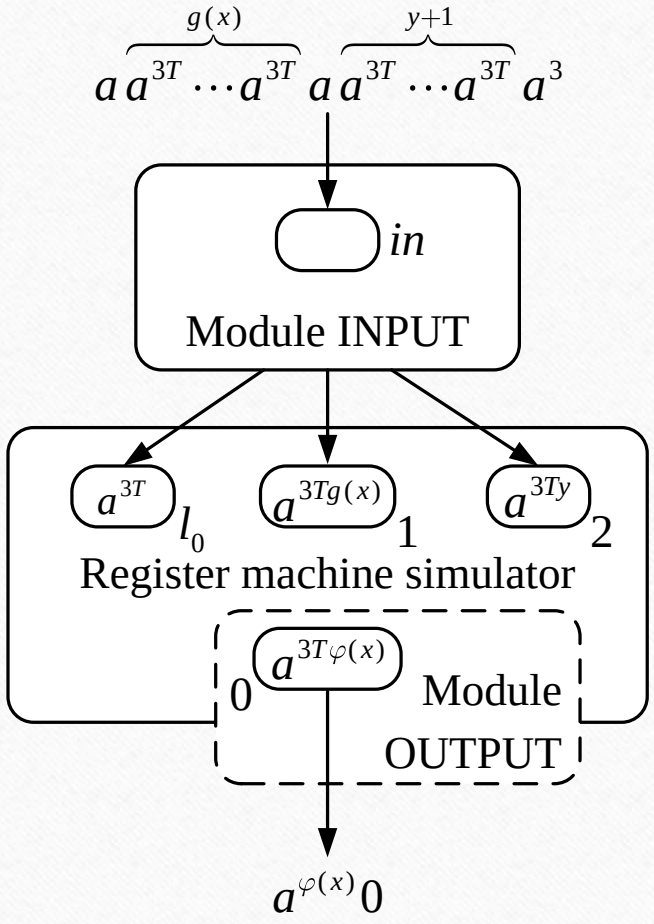
- An Example

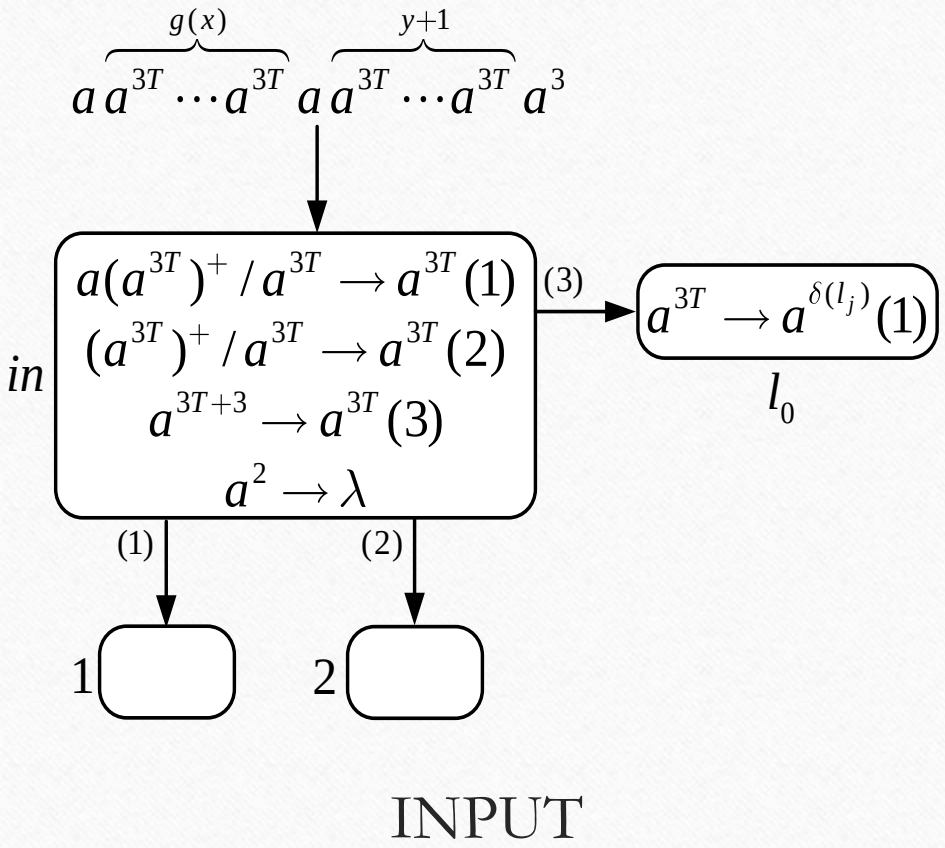


- Synchronous mode:
  - Work forever without output
- Asynchronous mode:
  - Output 1 spike at some step
  - Stop working at some step without output

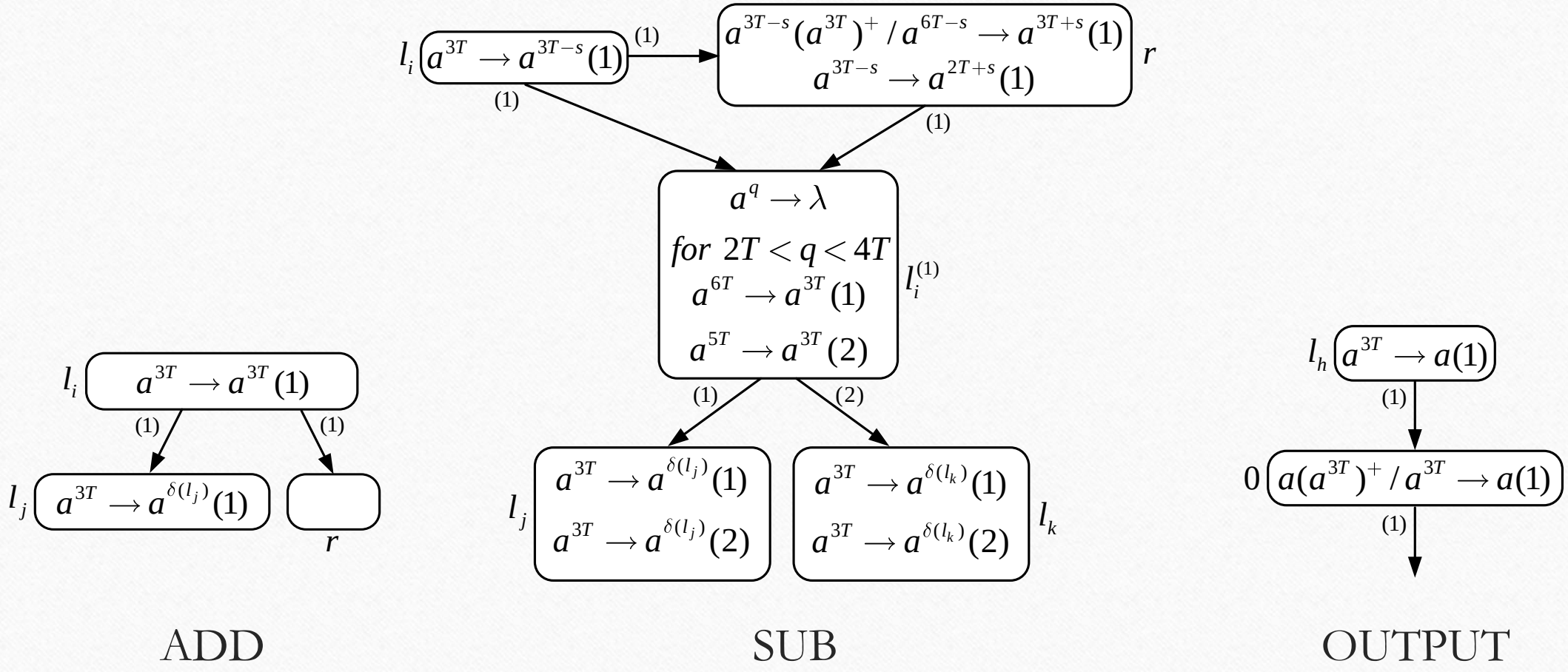
Asynchronous SNP-MC System for Computing Functions

The framework of the universal asynchronous SNP-MC system



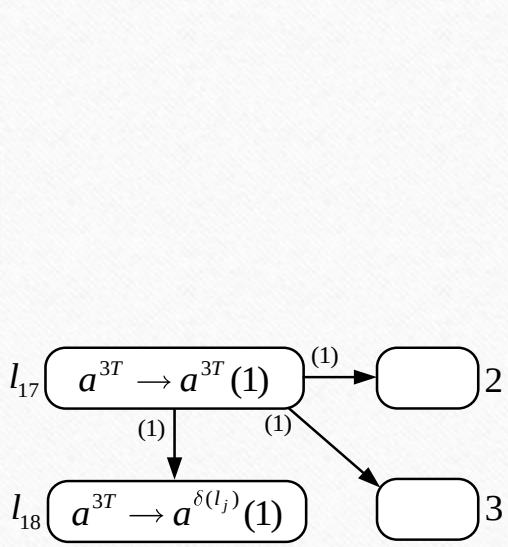


Asynchronous SNP-MC System for Computing Functions



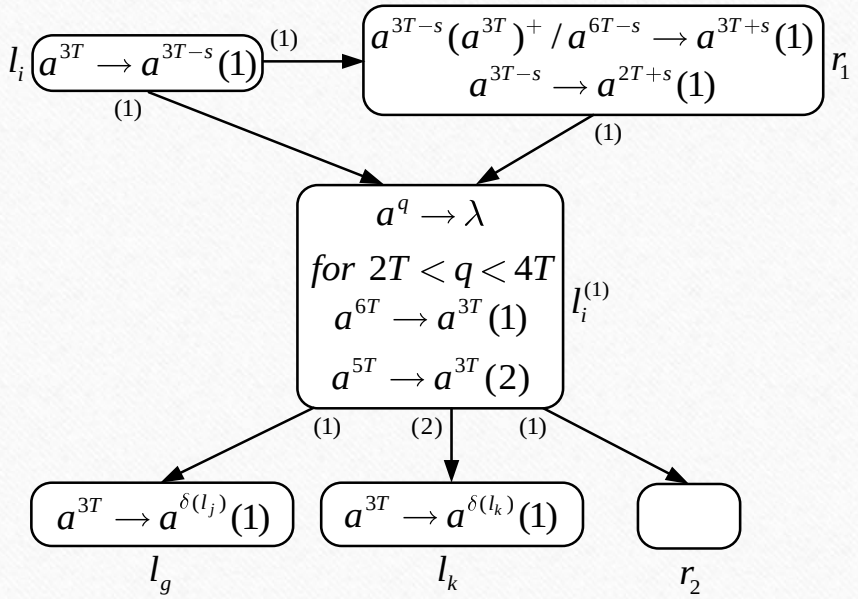
ADD SUB OUTPUT

Asynchronous SNP-MC System for Computing Functions



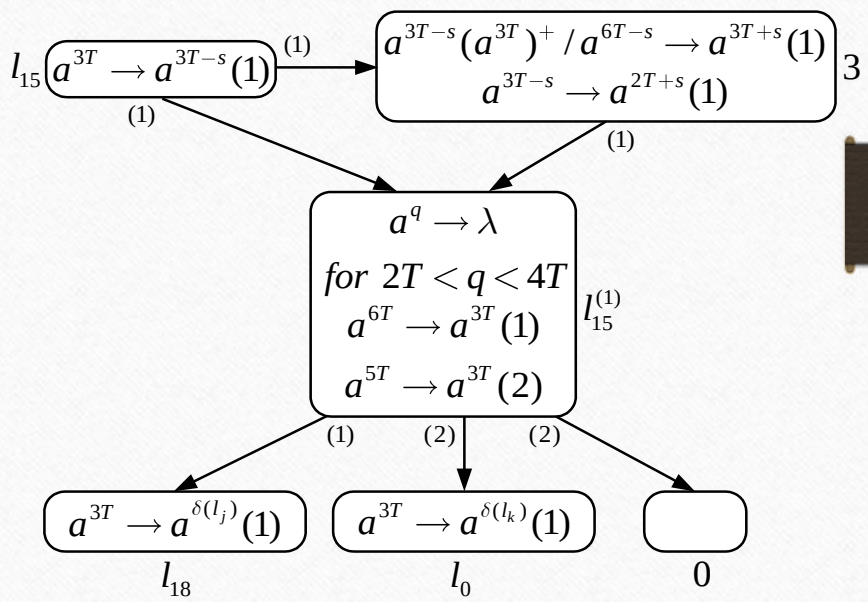
ADD-ADD

$l_{17} : (ADD(2), l_{21}), l_{21} : (ADD(3), l_{18})$



SUB-ADD-1

$l_i : (SUB(r_1), l_j, l_k), l_j : (ADD(r_2), l_g)$



SUB-ADD-2

$l_i : (SUB(r_1), l_j, l_k), l_k : (ADD(r_2), l_g)$

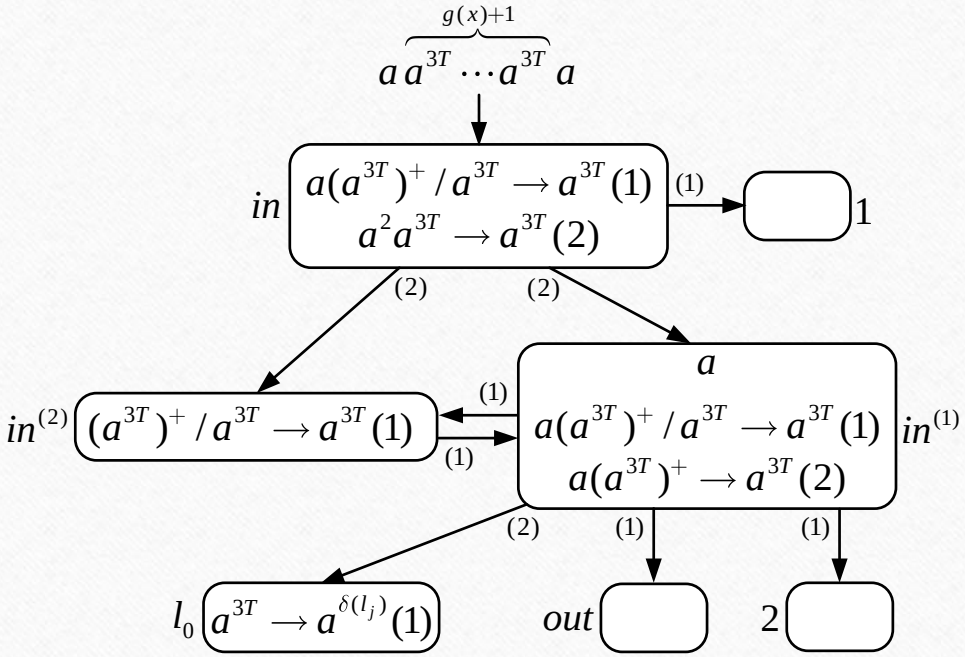
Asynchronous SNP-MC System for Computing Functions

- Asynchronous SNP-MC system Π_u has a total of 45 neurons:
  - 8 neurons: for the 8 registers,
  - 23 neurons: for the 23 labels,
  - 13 neurons: for neuron $\sigma_{li}^{(1)}$ in each SUB module, for a total of 13 SUB instructions,
  - 1 neuron: for neuron σ_{in} in the INPUT module.

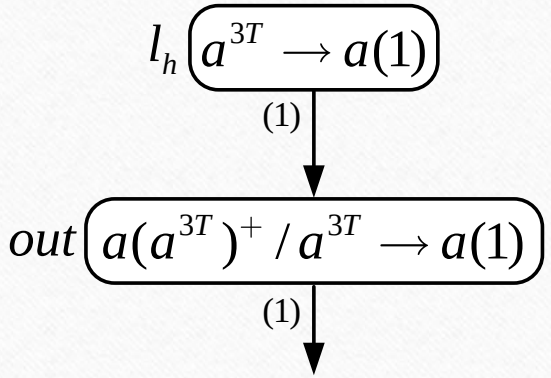
- From the preceding optimizations, we eliminate 7 neurons in total. The system needs only 38 neurons for computing functions.

Asynchronous SNP-MC System for Generating Numbers

- We redesign the INPUT module and OUTPUT module for system, but use the same ADD and SUB module as the above.



INPUT



OUTPUT

Asynchronous SNP-MC System for Generating Numbers

- 8 neurons: for the 8 registers,
- 16 neurons: for the 16 labels (l_{21} is saved by the module in Figure 8, l_1, l_5, l_7, l_9 and l_{16} are saved by the module in Figure 9, l_{20} is saved by the module in Figure 10),
- 13 neurons: for neuron $\sigma_{li}^{(1)}$ in each SUB module,
- 3 neurons: in INPUT module,
- 1 neuron: for neuron σ_{out} in OUTPUT module,
- which gives a total of 41 neurons.

Asynchronous SNP-MC System for Generating Numbers

Systems	Working model	Computing functions	Number generators
SN P system [7]	Synchronous	84	76
SN P system [46]	Synchronous	67	63
SN P system with weights [25]	Synchronous	48	45
SN P system with local synchronization [34]	Asynchronous	152	Unmeasured
SN P system [39]	Asynchronous	76	75
SN P system with rules on synapses [40]	Asynchronous	94	Unmeasured
SNP-MC system	Asynchronous	38	41

- Asynchronous SNP-MC system needs fewer neurons than all of the papers which have discussed small asynchronous SN P systems. Even then, asynchronous SNP-MC systems use less neuron than some of the SN P systems working in synchronous model. It shows that the multiple channels feature of our model plays a critical role in reducing the number of neurons.

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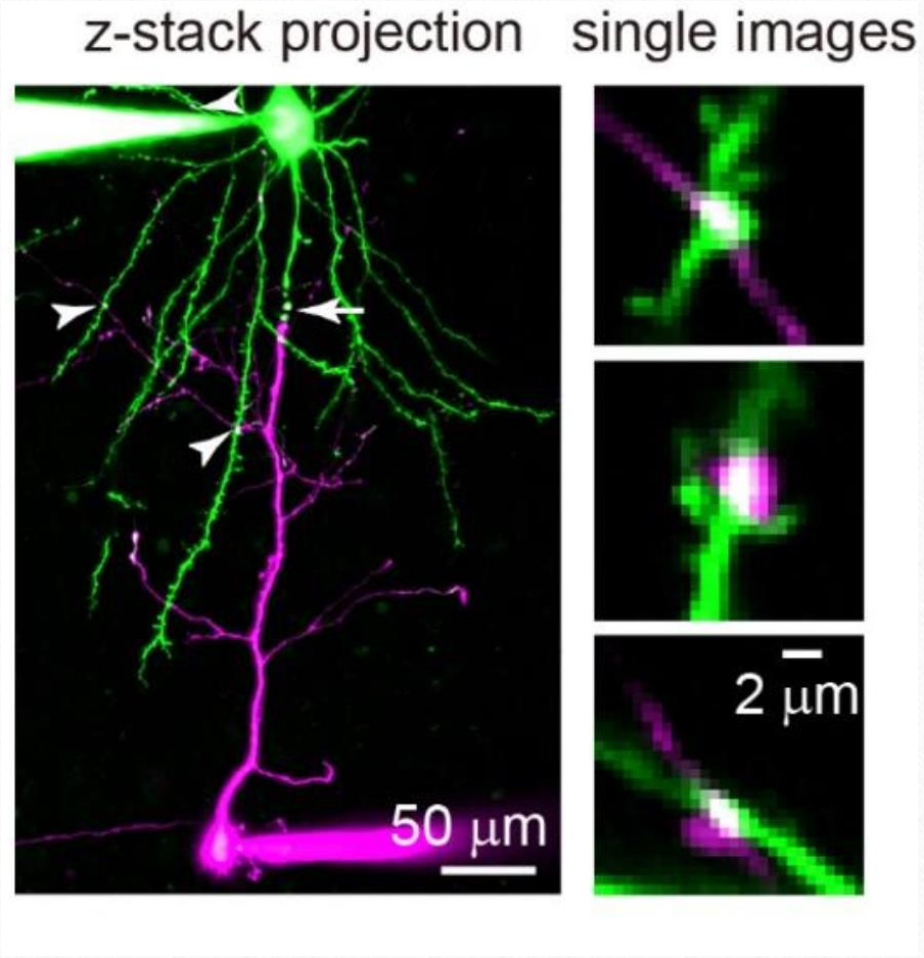
SN P Systems with Autapses

Autapses are special synapses that connect the axon and dendrites of a same neuron. They were considered to be redundant synaptic connections and seem unproductive in the last century. In recently, some experiments exposed that autapses play an important role in neural networks.



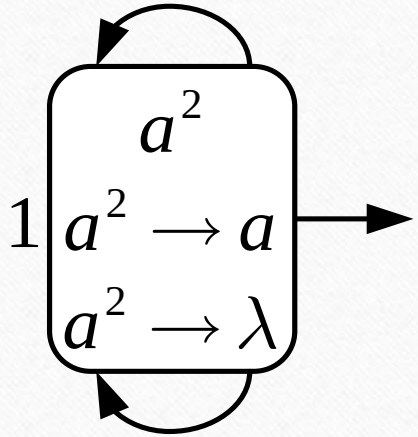
SN P Systems with Autapses[5]

- The first definition of SN P system given by Ionescu etc. in 2006 defined $(i,i) \notin syn$, Over the last ten years, these were hundreds of papers focus on SN P systems and all of them followed this assumption.
- Definition: $syn \subseteq \{1,2,...,m\} \times \{1,2,...,m\}$ with the form (i,j), for $1 \leq i, j \leq m$, which denotes the synapses.
- $2n$ spikes are used to represent number n in the registers in generating numbers, and $3n$ spikes are used to represent number n in the registers in computing functions.

[5] **Xiaoxiao Song** etc. Spiking Neural P Systems with Autapses. The draft is finished.

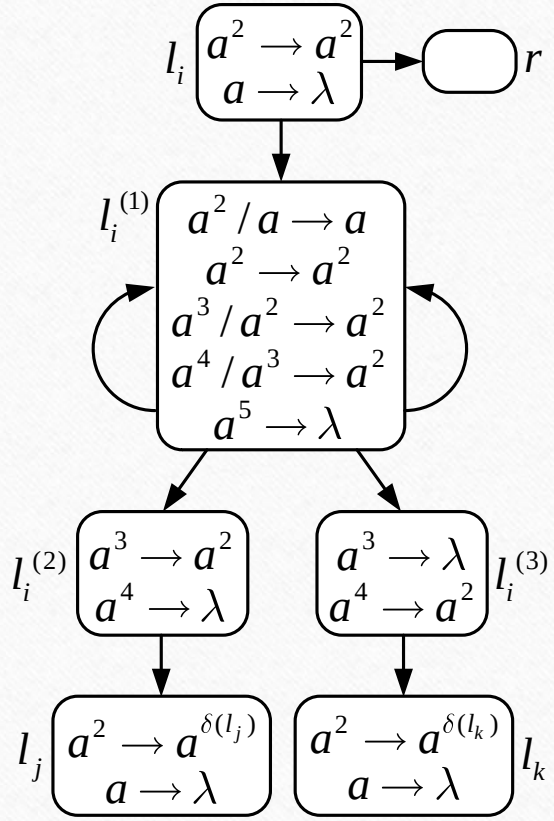
SN P Systems with Autapses

- An Example

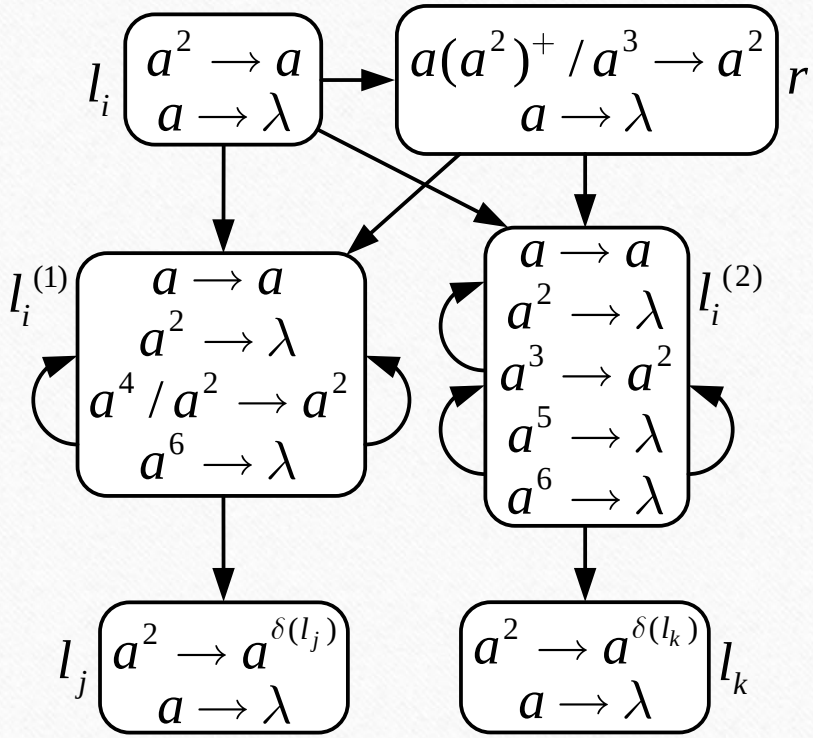


- Generate the set $\{n \mid n \geq 0\}$

SN P Systems with Autapses for Generating Numbers

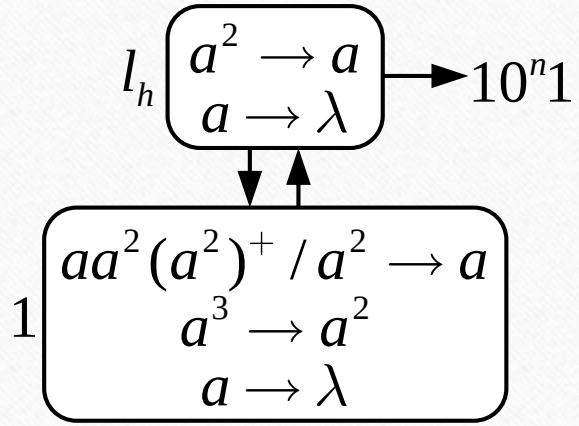


ADD



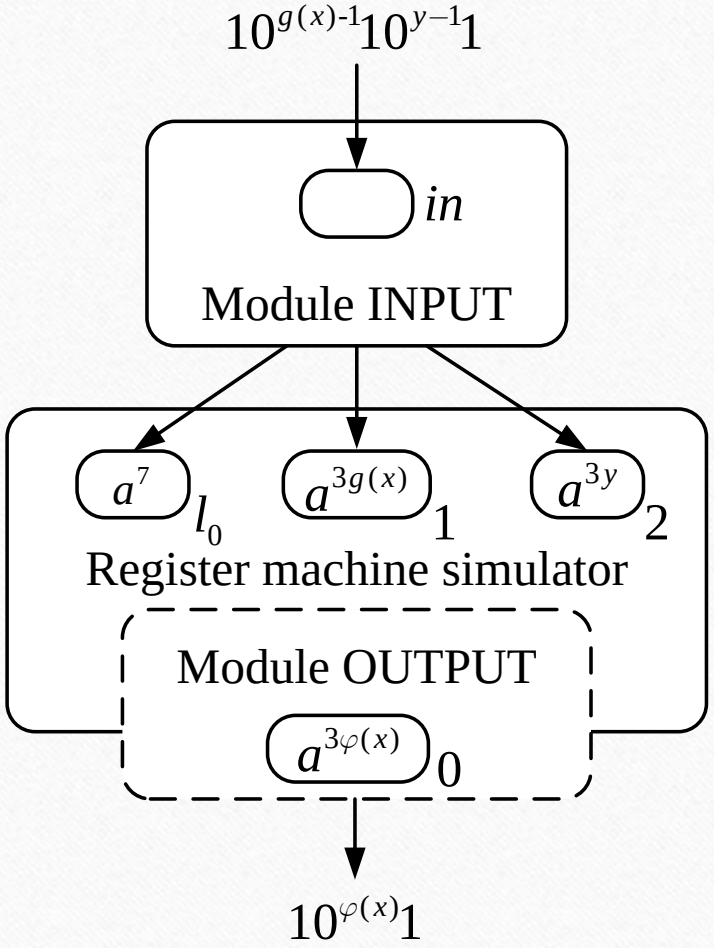
SUB

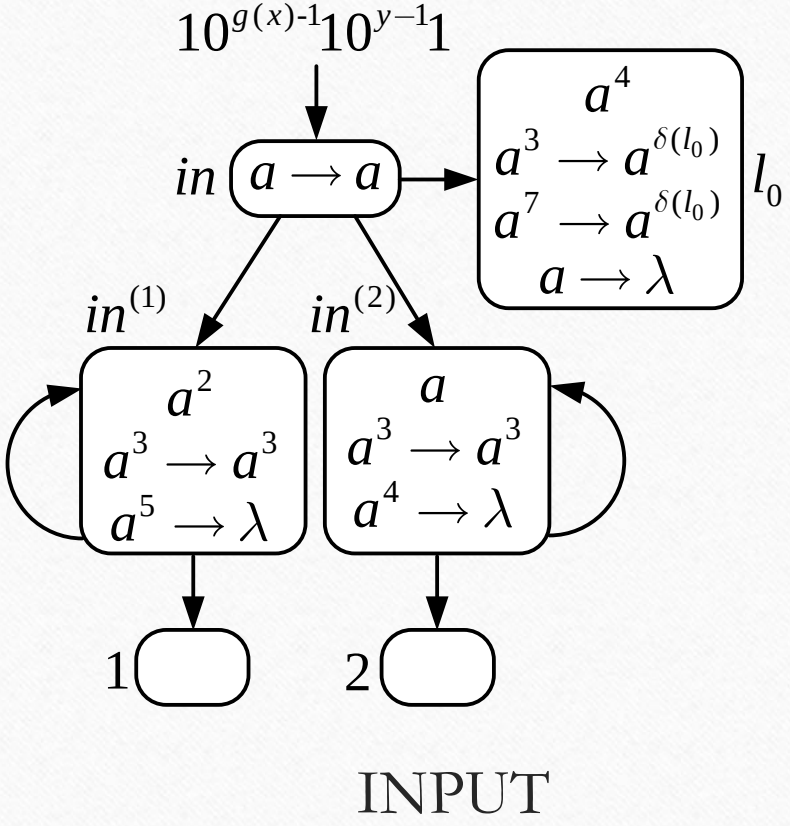
SN P Systems with Autapses for Generating Numbers



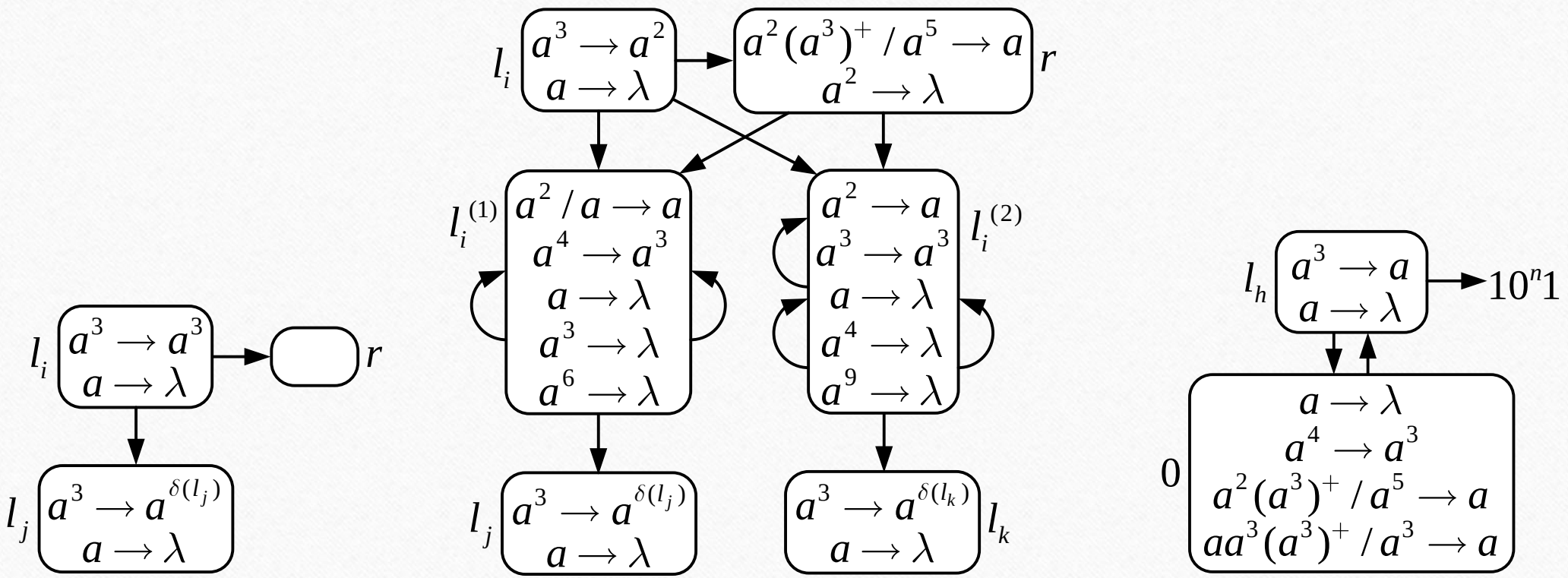
OUTPUT

SN P Systems with Autapses for Computing Functions

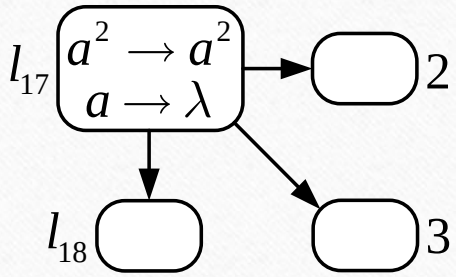




SN P Systems with Autapses for Computing Functions



SN P Systems with Autapses for Computing Functions



ADD-ADD

$l_{17} : (ADD(2), l_{21}), l_{21} : (ADD(3), l_{18})$

SN P Systems with Autapses for Computing Functions

- The system uses:
  - 8 neurons for 8 registers,
  - 23 neurons for 23 labels,
  - 2 auxiliary neurons in each SUB module, 26 in total,
  - 3 neurons in INPUT module,
- which comes to a total of 60 neurons.
- 1 neurons could be saved, and the number of neurons can be decremented from 60 to 59.

Future Works

- Only excitatory autapses are introduced into SN P systems, which means the autapses can lead new spikes back to the neuron and make the neuron excitatory again. **Inhibitory autapses** are found and play critical roles in regulating spike precision and network activity.
- Based on the latest research in autapses, when some autapses are placed properly in network, the **synchronization could be promoted** significantly. In this way, autapses could be introduced into asynchronous SN P systems to improve their performance.

THANK YOU FOR YOUR ATTENTION!

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2019.6